

Networks, Supply Chains, and Variational Inequalities

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Acknowledgments

I am deeply honored and very grateful for the opportunity to present to you today. Many thanks to the organizers for the invitation.



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Mathematics (QM) Information

Special acknowledgments and thanks to my collaborators and students who have made research and teaching always stimulating and rewarding.

- **Background and Inspiration**
- **Network Systems and the Braess Paradox**
- **Methodology - Variational Inequalities**
- **Representation of Supply Chains as Networks**
- **Food Supply Chains and Disruptions**
- **Cybercrime and Cybersecurity**
- **International Trade and Challenges**
- **The Multicommodity International Trade Model**
- **Making a Positive Impact**

Background and Inspiration

For the Love of **Operations Research**, a Branch of Applied Mathematics

When were you first captivated by Operations Research (OR)?

From my first university course on the subject to my first projects in industry - working in the high tech defense sector on naval submarines in Newport, Rhode Island, I was drawn to the power of the subject, especially when combined with computing.



Off to Grad School for a PhD

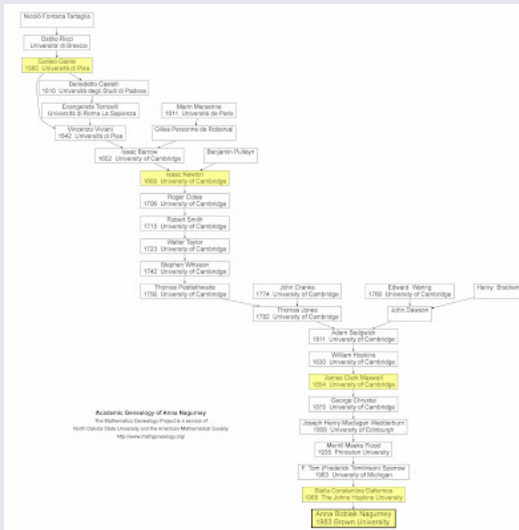
While working in high tech defense consulting I realized that I did not like having a boss. I commuted, ran marathons, and worked full time while taking courses for my Master's at Brown.

Dr. Stella Dafermos was the only female professor at the time in either Engineering or Applied Mathematics at Brown University. I became her first PhD student.



Stella was only the second female in the US to have received a PhD in OR and that was at Johns Hopkins University.

On the Shoulders of Giants - My Academic Genealogy - Maxwell, Newton, and Galileo



Network Systems and the Braess Paradox

I Work on the Modeling of Network Systems



And utilize optimization, game theory, and also dynamical systems to gain insights as to the behavior of stakeholders.

Some of My Books



Decentralized (Selfish) vs. Centralized (Unselfish) Behavior

User-Optimization (U-O) versus System-Optimization (S-O)



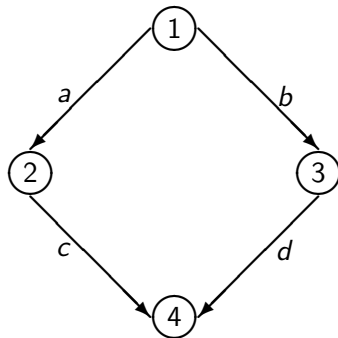
The Braess (1968) Paradox and User-Optimizing (U-O) Behavior

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of **6**, the equilibrium path flows are $x_{p_1}^* = x_{p_2}^* = 3$ and

The equilibrium path travel cost is

$$C_{p_1} = C_{p_2} = 83.$$



$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

Adding a Link Increases Travel Cost for All!

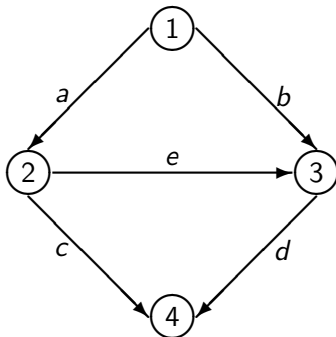
Adding a new link creates a new path $p_3 = (a, e, d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3 , $C_{p_3} = 70$.

The new equilibrium flow pattern network is

$$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2.$$

The equilibrium path travel cost: $C_{p_1} = C_{p_2} = C_{p_3} = 92$.



$$c_e(f_e) = f_e + 10$$

Travel cost / time increases for everyone!

“On a Paradox of Traffic Planning,” D. Braess, A. Nagurney, and T. Wakolbinger, *Transportation Science* 39 (2005), pp 446-450.

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The Braess Paradox Around the World

1969 - Stuttgart, Germany - The traffic worsened until a newly built road was closed.



1990 - Earth Day - New York City - 42nd Street was closed and traffic flow improved.



2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.





Braess

on

BROADWAY

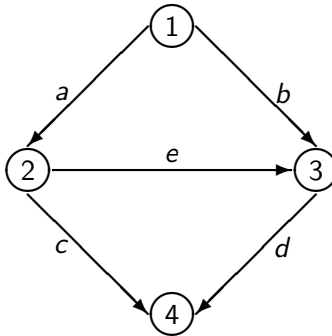


Interview on Broadway for *America Revealed*



Under S-O behavior, the total cost in the network is minimized, and the new route p_3 , under the same demand, would not be used.

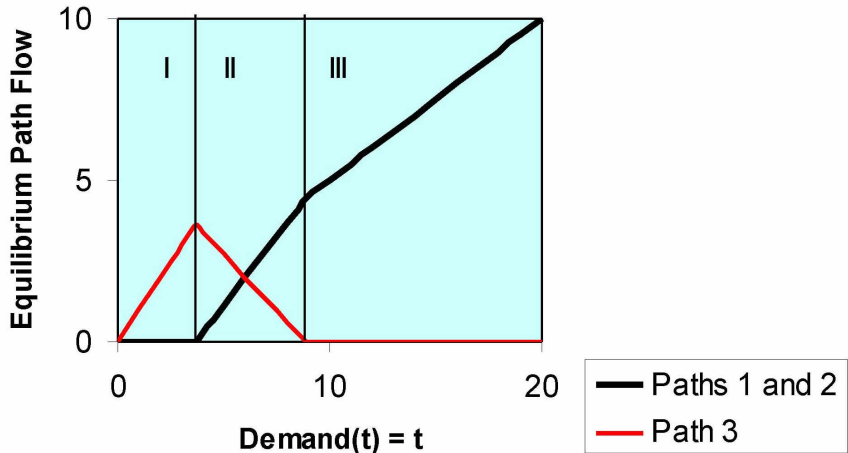
The Braess paradox never occurs in S-O networks.



Recall the Braess network with the added link e .

What happens as the demand changes?

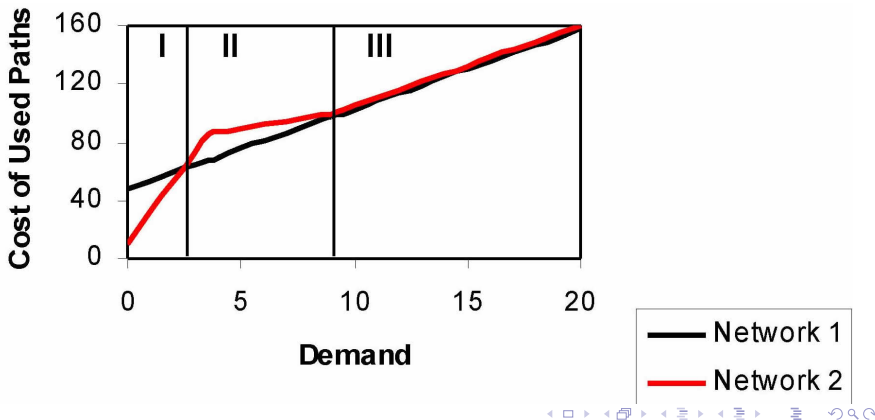
The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* (A. Nagurney, P. Daniele, and D. Parkes, *Computational Management Science* **4** (2007), pp 355-375).



In Demand Regime I, **Only the New Path is Used.**

In Demand Regime II, the travel demand lies in the range [2.58, 8.89], and **the Addition of a New Link (Path) Makes Everyone Worse Off!**

In Demand Regime III, when the travel demand exceeds 8.89, **Only the Original Paths are Used!**

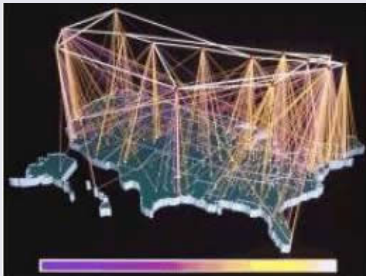


The new path is never used, under U-O behavior, when the demand exceeds 8.89, even when the demand goes out to infinity!



David Parkes, Patrizia Daniele, and Anna Nagurney at Harvard University.

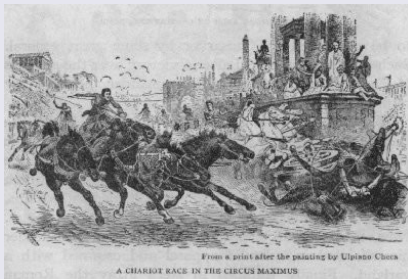
Other Networks that Behave like Traffic Networks



The Internet and electric power networks and even supply chains!

Congestion is Not a New Phenomenon

The study of the efficient operation of transportation networks dates to ancient Rome with a classical example being the publicly provided Roman road network and the time of day chariot policy, whereby chariots were banned from the ancient city of Rome at particular times of day.



Policies - Tolls and Congestion Pricing

Both William Vickrey, a Nobel laureate in Economic Sciences (in the 1950s), and Stella Dafermos (in the 1960s) with F. Tom Sparrow, constructed formulae for tolls, also, now known as **congestion pricing**.

Such policies have been implemented in several cities, including London and Gothenburg and now even in NYC, with the goal of reducing traffic congestion by altering the behavior of travellers / users of the transportation networks through pricing.

Methodology - Variational Inequalities

Variational Inequalities

Dafermos (1980) identified that the traffic network equilibrium conditions, as formulated by Smith (1979), were a variational inequality (VI) problem. This unveiled the theory for the formulation, analysis, and computation of solutions to numerous equilibrium problems in Operations Research, economics, engineering, and other disciplines.

The paper, available for free download, S. Dafermos (1980), “**Traffic Equilibrium and Variational Inequalities**,” *Transportation Science* 14(1), pp 42-54,



was selected by the Editors as one of the 12 most impactful papers in 50 years!

To-date, problems which have been formulated and studied as variational inequality (VI) problems include:

- traffic network equilibrium problems
- spatial price equilibrium problems
- oligopolistic market equilibrium problems
- financial equilibrium problems
- migration equilibrium problems, as well as
- environmental network and ecology problems,
- knowledge network problems,
- electric power generation and distribution networks,
- supply chain network equilibrium problems, and even
- the Internet!

Variational inequality theory provides us with a tool for:

- formulating a variety of equilibrium problems;
- qualitatively analyzing the problems in terms of existence and uniqueness of solutions, stability and sensitivity analysis, and
- providing us with algorithms with accompanying convergence analysis for computational purposes.

It contains, as special cases, such well-known problems in mathematical programming as: systems of nonlinear equations, optimization problems, complementarity problems, and is also related to fixed point problems.

Definition: Finite-Dimensional Variational Inequality Problem

The finite-dimensional variational inequality problem, $\text{VI}(F, \mathcal{K})$, is to determine a vector $X^ \in \mathcal{K} \subset R^{\mathcal{N}}$, such that*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

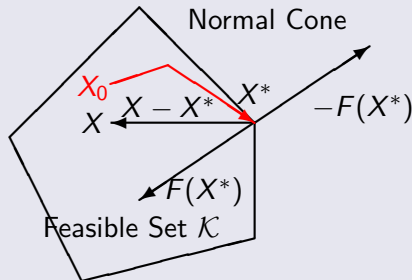
where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$ -dimensional Euclidean space. In the above inequality, $F(X) \equiv (F_1(X), F_2(X), \dots, F_{\mathcal{N}}(X))^T$, and $X \equiv (X_1, X_2, \dots, X_{\mathcal{N}})^T$.

The inequality is equivalent to

$$\sum_{i=1}^{\mathcal{N}} F_i(X^*) \cdot (X_i - X_i^*) \geq 0, \quad \forall X \in \mathcal{K}.$$

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System

As shown by Dupuis and Nagurney (1993), there is associated with a VI problem, a *projected dynamical system*, which provides a natural underlying dynamics until an equilibrium state is achieved, under appropriate conditions. In particular, $F(X^*)$ is “orthogonal” to the feasible set $\mathcal{K}$ at the point X^* .



To model the **dynamic behavior of complex network systems**, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in the *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are being used for **evolutionary games** (Sandholm (2005, 2011)), **ecological predator-prey networks** (Nagurney and Nagurney (2011a, b)), even **neuroscience** (Girard et al. (2008)), **dynamic spectrum model for cognitive radio networks** (Setoodeh, Haykin, and Moghadam (2012)), **Future Internet Architectures** (Saber, Nagurney, and Wolf (2014); see also Nagurney et al. (2015)).

Representation of Supply Chains as Networks

Much of My Recent Research Has Been on Supply Chains



Characteristics of Supply Chains and Networks Today

- **large-scale nature** and complexity of network topology;
- **congestion**, which leads to nonlinearities;
- **alternative behavior of users of the networks**, which may lead to paradoxical phenomena;
- **possibly conflicting criteria associated with optimization**;
- **interactions among the underlying networks themselves**, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;
- recognition of **their fragility and vulnerability**;
- policies surrounding networks today may have major impacts not only economically, but also **socially, politically, and security-wise**.

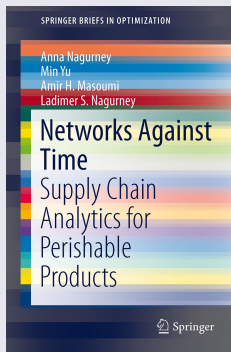
Representation of Supply Chains as Networks

By depicting supply chains as networks, consisting of nodes, links, flows (and also associated functions and behavior) we can:

- see **commonalities** and **differences** among supply chain problems and even other network problems;
- avail ourselves, once the underlying functions (cost, profit, demand, etc.), flows (product, informational, financial, relationship levels, etc.), and constraints (nonnegativity, demand, budget, etc.), and the behavior of the decision-makers is identified, of **powerful methodological network tools for modeling, analysis, and computations**;
- build meaningful extensions using the graphical/network conceptualization.

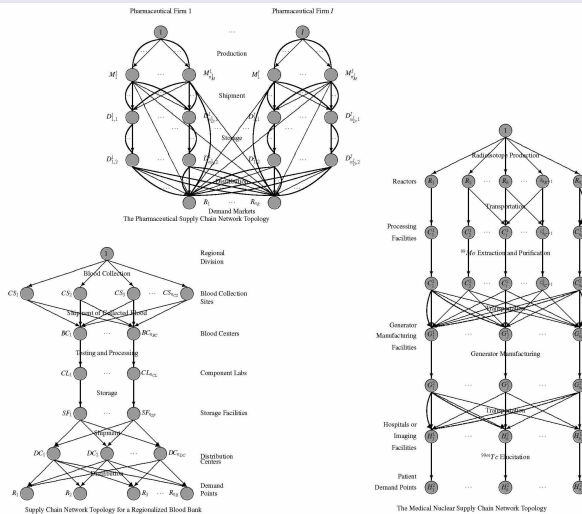
A Multidisciplinary Approach

In our research on perishable and time-sensitive product supply chains, we utilize results from physics, chemistry, biology, and medicine in order to capture the perishability of various products over time from healthcare products such as blood, medical nucleotides, and pharmaceuticals to food.

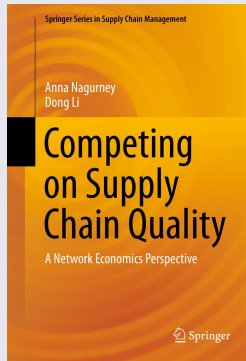


Some of the Supply Chain Network Topologies

Applications to pharmaceutical supply chains, blood and medical nuclear ones, and, of course, food.

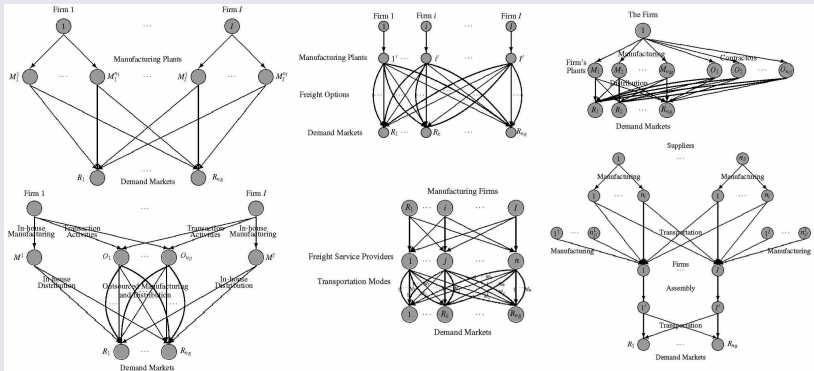


Research on Quality is Related to That on Perishability



Examples of product quality failures have included: •
adulterated infant formula • inferior pharmaceuticals •
defective airbags • defective ignition switches •
bacteria-laden food • exploding smartphones, etc.

In the book, we present supply chain network models and tools to investigate, amongst other topics, information asymmetry, impacts of outsourcing on quality, minimum quality standards, applications to industries such as pharma, freight services and quality, and **the identification of which suppliers matter the most to both individual firms' supply chains and to that of the supply chain network economy.**



Food Supply Chains and Disruptions

Food Supply Chains

Food is essential to our health and well-being. During the Covid-19 pandemic, declared on March 11, 2020 by the World Health Organization, the associated supply chains suffered major disruptions. Various disruptions continue because of climate change, Russia's war on Ukraine, and other disasters.



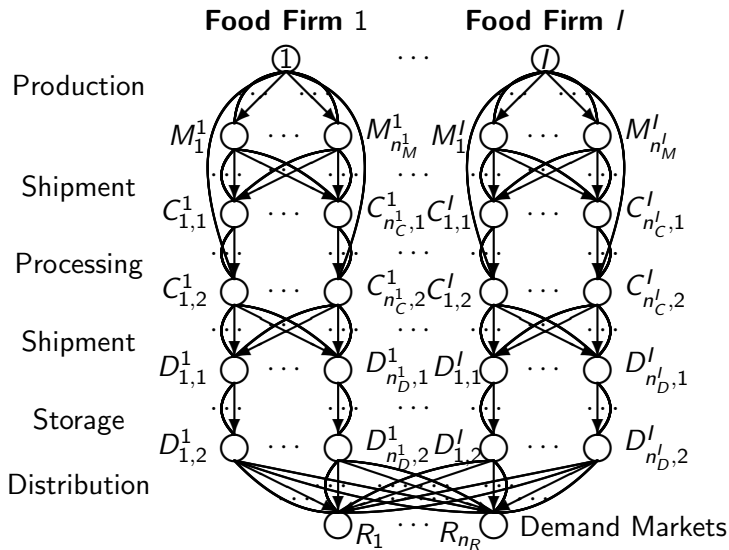
Fresh Produce Food Supply Chains

Our fresh produce supply chain network oligopoly model:

- ① captures the deterioration of fresh food along the entire supply chain from a network perspective;
- ② handles the time decay through the introduction of arc multipliers;
- ③ formulates oligopolistic competition with product differentiation;
- ④ includes the disposal of the spoiled food products, along with the associated costs;
- ⑤ allows for the assessment of alternative technologies involved in each supply chain activity.

M. Yu and A. Nagurney, “Competitive Food Supply Chain Networks with Application to Fresh Produce,” *European Journal of Operational Research* 224(2) (2013), pp 273-282.

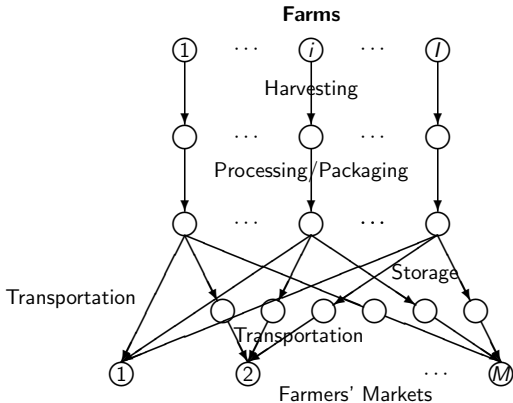
Fresh Produce Food Supply Chains



The Fresh Produce Supply Chain Network Topology

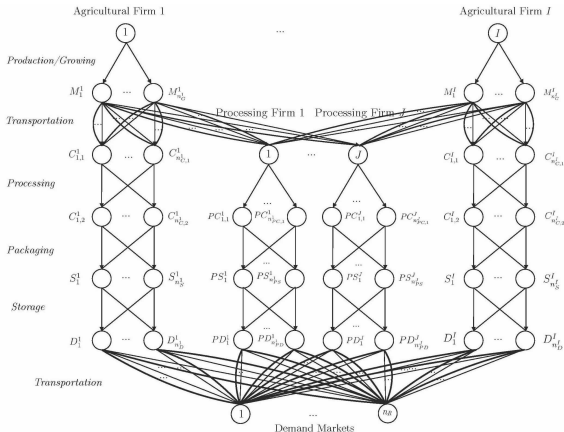
Farmers' Markets and Fresh Produce Supply Chains

- The I farms compete **noncooperatively** in an **oligopolistic** manner.
- Products are differentiated based on **quality** at the farmers' markets.



D. Besik and A. Nagurney, "Quality in Competitive Fresh Produce Supply Chains with Application to Farmers' Markets," *Socio-Economic Planning Sciences* 60 (2017), pp 62-76.

Integrated Supply Chain Network Model



D. Besik, A. Nagurney, and P. Dutta, "An Integrated Multitiered Supply Chain Network Model of Competing Agricultural Firms and Processing Firms: The Case of Fresh Produce and Quality,"
European Journal of Operational Research 307(1) (2023), pp 364-381.

Food Supply Chain Disruptions Due to COVID-19



AMERICA'S FOOD CHAIN

As coronavirus pandemic spikes orange juice sales, a Florida citrus grower gets squeezed

Janine Zeitlin, USA TODAY Network - Florida
Updated 8:07 p.m. EDT May 14, 2020

QW

An Idaho farm is giving away 2 million potatoes because coronavirus has hurt demand



By Alisha Ebrahimji, CNN
Updated 1:33 PM ET, Thu April 16, 2020



Lacking seasonal workers, Italy elevates its long-shunned migrants

THE CHRISTIAN SCIENCE
MONITOR



Farms encountering guest worker shortage amid new coronavirus restrictions

REUTERS

Piglets aborted, chickens gassed as pandemic slams meat sector

The Washington Post
Democracy Dies in Darkness

The meat industry is trying to get back to normal. But workers are still getting sick – and shortages may get worse.

There are now more than 11,000 coronavirus cases tied to Tyson Foods, Smithfield Foods and JBS

Germany Struggles To Fill Its Farm Labor Shortage After Closing Its Borders

May 20, 2020 - 10:58 AM ET



ROB SCHMITZ

n p r

It's All About People

A major research theme of ours in the COVID-19 pandemic (which continues) was the inclusion of labor in supply chains, using optimization and game theory.



January 29, 2021 in [Supply Chain Networks](#)

In the End, It's All About People

COVID-19 vaccine production reveals dependency on supply chains, labor workforce in the U.S.

By Anna Nagurney

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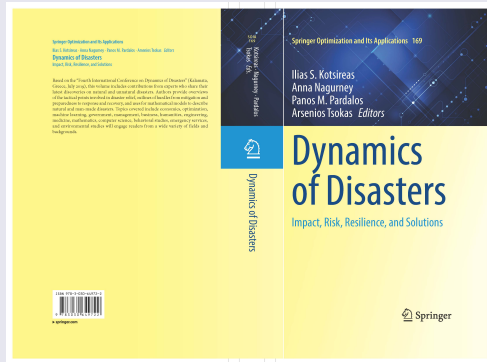
PRINT ARTICLE: [📄](#)

<https://doi.org/10.1287/orms.2021.01.17>



The COVID-19 pandemic has dramatically revealed how dependent we are on supply chains and the availability of labor. Without the human element, meatpacking plants cannot function; fresh produce cannot be picked; grocery stores cannot be shelled; PPEs cannot be produced and distributed; and products cannot be delivered to our homes through e-commerce. Also, COVID-19 vaccine production may lack the human resources to ensure product quality.

“Perishable Food Supply Chain Networks with Labor in the Covid-19 Pandemic,” A. Nagurney, in: *Dynamics of Disasters - Impact, Risk, Resilience, and Solutions*, I.S. Kotsireas, A. Nagurney, P.M. Pardalos, and A. Tsokas, Editors, Springer Nature Switzerland AG, 2021, pp 173-193.



Perishable Food Supply Chain Network Model with Labor

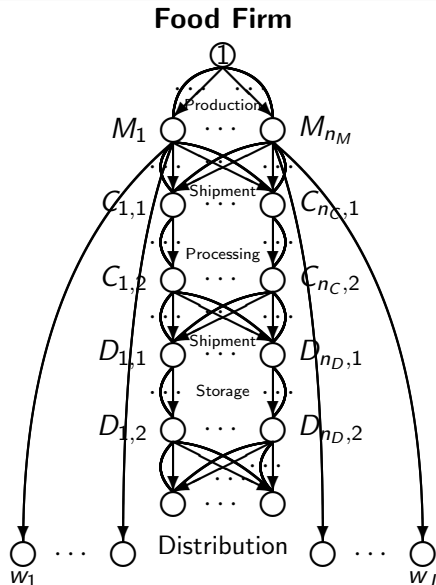


Figure: The Perishable Food Supply Chain Network Topology

Our findings include:

- ① The lack of labor on a single link, even a freight one, may significantly negatively impact a food firm.
- ② Preserving productivity in all utilized supply chain network economic activities is critical since the impact of a drastic reduction can severely reduce profits.
- ③ Adding more direct sales, whether at farmers' markets or nearby farm stands, may help a food firm in a pandemic.
- ④ Also, if a firm enhances its marketing so as to have consumers be willing to pay a higher price for its fresh produce, major profit increases can occur.

In “Supply Chain Game Theory Network Modeling Under Labor Constraints: Applications to the Covid-19 Pandemic,” **A. Nagurney**, *European Journal of Operational Research* **293(3)**, (2021), pp 880-891, a game theory model for supply chains with labor was constructed, under three different sets of constraints, building on our previous work.

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European Journal of Operational Research

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2021

Presented to

Anna Nagurny

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Prof. Roman Slowinski
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Game Theory Supply Chain Network Model with Labor

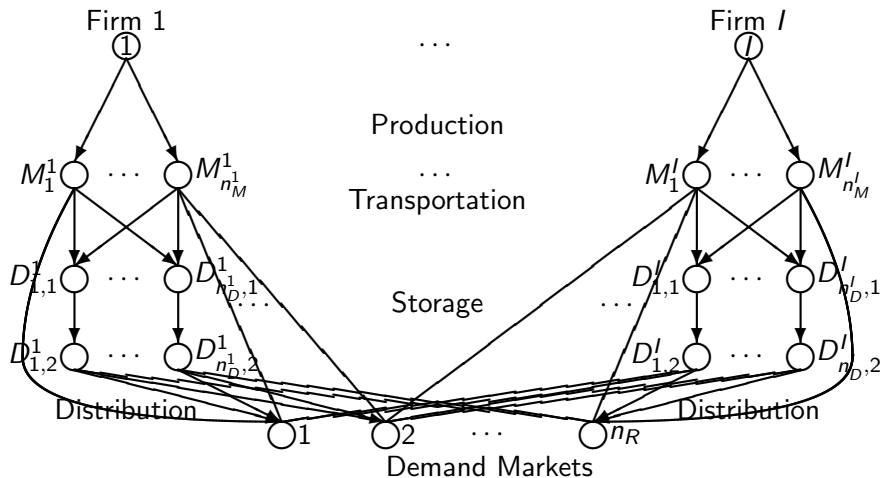


Figure: The Supply Chain Network Topology of the Game Theory Model with Labor

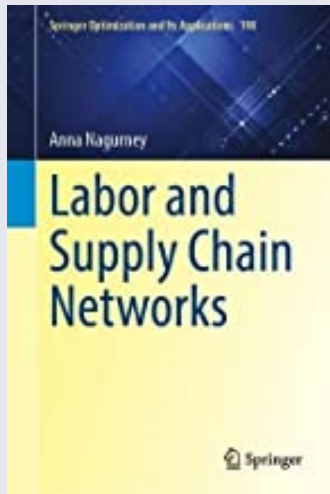
Numerical Experiments

Our numerical examples are based on disruptions in migrant labor in the blueberry supply chain in the Northeast of the US in the summer of 2020.

- Disruptions in labor on a supply chain network link;
- Addition of a competitor;
- Modifications in demand price functions;
- Sensitivity analysis in terms of labor availability.

The full input and out data are available in our paper in the *European Journal of Operational Research*.

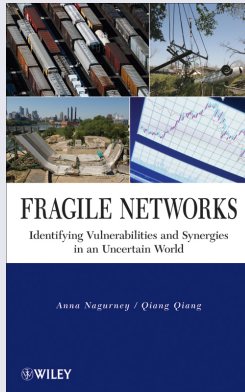
Farmers should do everything possible to secure the health of the workers at his production/harvesting facilities, so that the blueberries can be harvested in a timely manner and so that profits do not suffer. Keeping workers healthy, through appropriate measures, impacts the bottom line!



Cybercrime and Cybersecurity

How I Became Interested in Cybersecurity

One of my books, written with a UMass Amherst Isenberg School PhD alum, was “hacked” and digital copies of it posted on websites around the globe.



In a sense, this may be viewed as a compliment since clearly someone had determined that it has some sort of value. ➤

Cybercrime and Cybersecurity

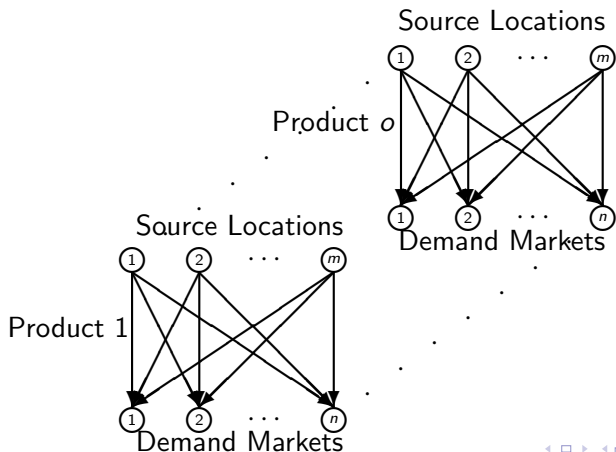
The publisher John Wiley & Sons was notified and lawyers got involved but how do you contact and then influence those responsible for postings on rather anonymous websites?

Clearly, hackers go where there is money.

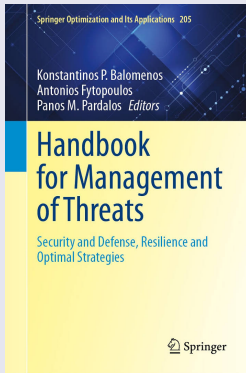


Perishability and Cybercrime in Financial Products

The paper, “A Multiproduct Network Economic Model of Cybercrime in Financial Services,” A. Nagurney, *Service Science* 7(1) (2015) pp 70-81 provides insights into the perishability of value of credit cards.



Another Relevant Book and Chapter of Ours



Defense-Critical Supply Chain Networks and Risk Management with the Inclusion of Labor: Dynamics and Quantification of Performance and the Ranking of Nodes and Links

Anna Nagurny

1 Introduction

In February 2022, the U.S. Department of Defense (DoD) issued a long-awaited report, "Securing Defense-Critical Supply Chains" (U.S. Department of Defense [20]). The report was in response to Executive Order (EO) 14037, "America's Supply Chains," signed by President Joseph R. Biden Jr., to identify how to improve supply chain resilience and how to protect against material shortages, which had clearly become exacerbated in the COVID-19 pandemic (see Biden Jr. [1], United States White House [20]). The DoD's report provided an assessment of defense-critical supply chains in order to improve the department's capacity to defend the United States. With the geopolitical risk rising globally and, with the war of Russia against Ukraine raging off, Brankley, Peters-Pere, and Nagurny and Litnagel [21], following the major invasion, beginning February 24, 2022, having a framework for the modeling, analysis, and solution of defense-critical supply chains is of major importance. Of additional relevance is having a framework to identify which of the nodes and links, corresponding, for example, to manufacturing sites and processes, storage facilities, transportation and distribution, are important sites focusing on those can help to preserve the performance of the supply chain networks for critical defense products in the case of disruptions.

Precursor to the COVID-19 pandemic, which is a global healthcare disaster, not limited in location or in a time window, the number of disasters, including "natural" disasters, has been growing as well as the people affected by them (see Nagurny and Qing [18] and Katsirelos et al. [19]). Hence, research on supply chain network

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K. P. Balomenos et al. (eds.), *Handbook for Management of Threats*, Springer
Optimization and Its Applications 205,
https://doi.org/10.1007/978-3-031-39842-8_3

International Trade and Challenges

International Trade

International trade provides us with commodities throughout the year and has benefits for producers and consumers alike.



World's Biggest Importers

Largest Importers In The World



In 2022, global imports climbed to \$25.6 trillion in value, or about the size of the U.S. GDP. As an engine of growth, global trade broadens consumer choices and can lower the cost of goods. For businesses, it can improve the quality of inputs and strengthen competitiveness.

*(in Billion USD)

	United States	\$3,376B
	China	\$2,716B
	Germany	\$1,571B
	Netherlands	\$899B
	Japan	\$897B
	United Kingdom	\$824B
	France	\$818B
	South Korea	\$731B
	India	\$723B
	Italy	\$689B
	Hong Kong SAR	\$668B
	Mexico	\$626B
	Belgium	\$621B
	Canada	\$582B
	Spain	\$493B
	Singapore	\$476B
	Taiwan	\$436B
	UAE	\$425B
	Poland	\$381B
	Türkiye	\$364B

	VietNam	\$359B
	Switzerland	\$356B
	Australia	\$309B
	Thailand	\$303B
	Malaysia	\$294B
	Brazil	\$290B
	Russia	\$242B
	Indonesia	\$237B
	Czech Republic	\$236B
	Austria	\$232B
	Sweden	\$202B
	Saudi Arabia	\$188B
	Hungary	\$163B
	Ireland	\$146B
	Philippines	\$144B
	South Africa	\$136B
	Romania	\$132B
	Denmark	\$127B
	Portugal	\$115B
	Slovakia	\$113B

Source: World Trade Organization, USAFacts
www.rankingroyals.com

RankingRoyals



World Trade Data - Top Exporters and Importers



Global Trade Policies

Examples of policy instruments that have been applied by governments to modify trade patterns include: **tariffs**, **quotas**, and a combination thereof - **tariff rate quotas**.



Tariffs Are Regularly in the News!

The imposition of tariffs by certain countries is leading to retaliation by other countries with ramifications across multiple supply chains, and a **trade war**.

With Higher Tariffs, China Retaliates Against the U.S.



The Yangshan Deep Water Port in Shanghai, China. The Chinese government said on Monday that it would raise tariffs on goods from the United States as of June 1, giving negotiators from the two countries time to strike a deal. *Aly Song/Reuters*

Trump's Tariffs Would Deal a Big Blow to the Auto Industry

Automakers and parts suppliers would struggle if President-elect Donald J. Trump followed through on his threat to impose 25 percent tariffs on imports from Canada and Mexico.



Nissan and other automakers are cutting thousands of jobs as they struggle to cope with sagging demand and a growing preference for hybrid and electric cars. *Ueno/Reuters*

President-elect Donald J. Trump's threat to impose 25 percent tariffs on goods from Mexico and Canada sent shivers on Tuesday through the auto industry, which depends heavily on both countries for parts and manufacturing.

We have been developing **computable operational mathematical models** that enable the assessment of the impacts of trade policy instruments such as tariff rate quotas on consumer prices, trade flows, as well as on the profits of producers/firms.

This is very challenging research!

Motivation

- A tariff rate quota (TRQ) is a **two-tiered tariff**, in which a lower **in-quota tariff** is applied to imports until a quota is attained and then a higher **over-quota tariff** is applied to all subsequent imports.
- The Uruguay Round in 1996 induced the creation of more than 1,300 new TRQs.
- The world's four most important food crops: rice, wheat, corn, and bananas have all been subject to tariff rate quotas.



Some of Our Papers on Tariffs and Other Policies



International Agricultural Trade

International agricultural trade provides us with essential agri-food commodities throughout the year, ensuring our food security and simultaneously benefiting the farmers.



An Example of Our Trade Policy Research

A. Nagurney, D. Besik, and L.S. Nagurney, “Global Supply Chain Networks and Tariff Rate Quotas: Equilibrium Analysis with Application to Agricultural Products, *Journal of Global Optimization* 75 (2019), pp 439-460.



The model is a game theory one with tariff rate quotas and the numerical case study is on avocados.

Another Example of Our Trade Policy Research

A. Nagurney, D. Besik, and J. Dong, “Tariffs and Quotas in World Trade: A Unified Variational Inequality Framework,”
European Journal of Operational Research 275(1) (2019), pp 347-360.



The model is a spatial price equilibrium one with tariff rate quotas and the numerical case study is on the dairy industry.

Investigation of Tariffs and Quotas in the Pandemic

A. Nagurney, M. Salarpour, and J. Dong, “Modeling of Covid-19 Trade Measures on Essential Products: A Multiproduct, Multicountry Spatial Price Equilibrium Framework,” *International Transactions in Operational Research* 29(1) (2022), pp 226-258.



The model is a spatial price equilibrium one with unit tariffs, quotas, price floors and ceilings. The case study is on PPEs.

Disasters and Food Security

- Climate change and COVID-19 impacted the affordability and accessibility of agri-food products around the globe.
- With the added disruptions of Russia's full-scale invasion of Ukraine on February 24, 2022, around 47 million people are estimated to have been added to the more than 276 million who were already facing food insecurity.
- Critical links such as the Panama Canal, the Red Sea and Suez Canal, and the Strait of Hormuz have been disrupted, respectively, because of a drought affecting the Panama Canal, Houthi attacks, and the US war on Iran.



UMass Amherst and KSE Partnership

In March 2022, UMass Amherst and KSE established a global partnership.



The partnership includes a Virtual Scholar in Residence Program and exchange students from KSE.

Acknowledgment

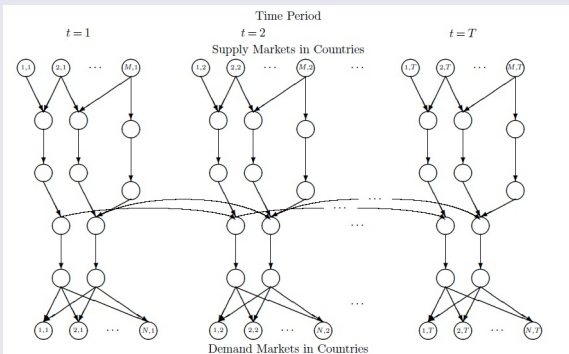


I acknowledge the partnership between the University of Massachusetts Amherst and the Kyiv School of Economics, which facilitated our research on international agricultural trade.

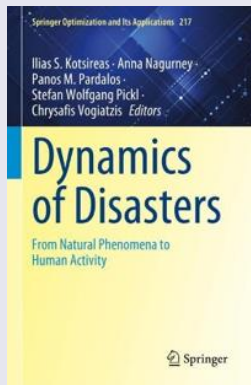


Networks and Supply Chains

A Multiperiod International Agricultural Trade Network Topology



D. Hassani, A. Nagurney, O. Nivievskiy, and P. Martyshev, “A Multiperiod, Multicommodity, Capacitated International Agricultural Trade Network Equilibrium Model with Applications to Ukraine in Wartime,” *Transportation Science* **59**(1) (2025), pp 143-164.



In the edited volume is the paper, “Quantification of International Trade Network Performance Under Disruptions to Supply, Transportation, and Demand Capacity, and Exchange Rates in Disasters,” by A. Nagurney, D. Hassani, O. Nivievskyi, and P. Martyshev.

The Multicommodity International Trade Model

The Multicommodity International Trade Model

A. Nagurney, D. Hassani, O. Nivievskiy, and P. Martyshev,
“Exchange Rates and Multicommodity International Trade:
Insights from Spatial Price Equilibrium Modeling with Policy
Instruments via Variational Inequalities,” *Journal of Global
Optimization* 87 (2023), pp 1-30.



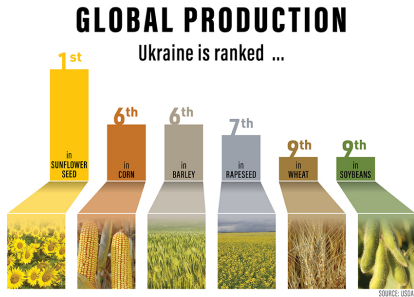
The Multicommodity International Trade Model

In this paper, we harness the powerful theory of variational inequalities to construct a model with the following features:

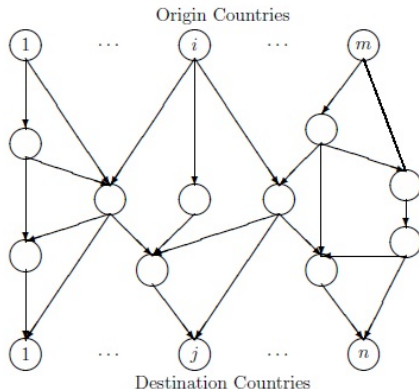
- **Multiple commodities;**
- **Multiple routes from origin nodes to destination nodes** in the same or different countries;
- **Exchange rates** and the formula for their computation along trade routes;
- Inclusion of policies in the form of **tariffs, subsidies, and quotas;**
- The underlying **economic functions can be nonlinear and asymmetric**. Hence, our **transportation cost functions capture congestion**.

Additional Motivation

This work is inspired by Russia's war on Ukraine and the need to assess its impacts on agricultural trade as well as on food insecurity and the need to provide more general computable models for assessing the impacts of trade policies and exchange rates on international trade.



The Multicommodity International Trade Model



The network topology is denoted by the graph $G = [N, L]$, where N is the set of nodes, L is the set of links, and P is the set of paths. There are H commodities, with a typical one denoted by h .

The Multicommodity International Trade Model

Each path p represents a trade route. Intermediate nodes in the network, which are transit points, also correspond to countries.

Let P_{ij} denote the set of paths connecting the pair of origin/destination country nodes (i, j) . The paths are acyclic.

A typical link is denoted by a and represents transport from a country node at which the link originates to the node denoting the country at which the link terminates.

A trade route can entail transportation through multiple countries, depending on the application, and via different modes, such as rail, truck, air, or water (sea, river, etc.).

The Exchange Rates

Associated with each link $a \in L$ is an exchange rate e_a , reflecting the exchange rate from the country (node) that the link emanates from to the country (node) that it terminates in.

Also, associated with each pair of origin/destination countries (i, j) is the exchange rate e_{ij} for $i = 1, \dots, m; j = 1, \dots, n$.

The Trade Policies

There is a nonnegative subsidy associated with commodity h and imposed by the government in country i , which is denoted by sub_i^h for $h = 1, \dots, H$; $i = 1, \dots, m$.

The unit tariff levied by country j on commodity h from country i is denoted by τ_{ij}^h for $h = 1, \dots, H$; $i = 1, \dots, m$; $j = 1, \dots, n$.

Tariffs within a country are not imposed; hence, $\tau_{ii}^h = 0$, $\forall i, \forall h$.

In addition, there are capacities, which can represent quotas, where $\bar{Q}_p^h$; $h = 1, \dots, H$; $p \in P$, denotes the bound on the commodity shipment of commodity h on path p .

Variables and Constraints

All commodity path flows, for all commodities h , and all paths p , must be nonnegative:

$$Q_p^h \geq 0, \quad \forall h, \forall p \in P. \quad (1)$$

The flow on a link a of commodity h , in turn, is equal to the sum of the path flows of the commodity h that use the link:

$$f_a^h = \sum_{p \in P} Q_p^h \delta_{ap}, \quad \forall h, \forall a \in L. \quad (2)$$

where $\delta_{ap} = 1$, if link a is contained in path p , and is 0, otherwise.

Variables and Constraints

The supply of commodity h produced in country i , s_i^h , is equal to the shipments of the commodity from the country to all destination countries:

$$s_i^h = \sum_{p \in P^i} Q_p^h, \quad h = 1, \dots, H; i = 1, \dots, m, \quad (3)$$

whereas the demand for commodity h in country j , d_j^h , is equal to the shipments of the commodity from all origin countries to that country:

$$d_j^h = \sum_{p \in P_j} Q_p^h, \quad h = 1, \dots, H; j = 1, \dots, n. \quad (4)$$

$Q \in R_+^{np}$ is the vector of commodity shipments with $s \in R_+^{Hm}$ being the vector of commodity supplies and $d \in R_+^{Hn}$ being the vector of commodity demands.

Supply Price Functions

The supply price function for commodity h of country i is denoted by π_i^h and we have that:

$$\pi_i^h = \pi_i^h(s), \quad h = 1, \dots, H; i = 1, \dots, m. \quad (5a)$$

With notice of the conservation of flow equations (3), we may define new supply price functions $\tilde{\pi}_i^h$; $h = 1, \dots, H$; $i = 1, \dots, m$, such that

$$\tilde{\pi}_i^h(Q) \equiv \pi_i^h(s). \quad (5b)$$

Demand Price Functions

The demand price functions, in turn, are:

$$\rho_j^h = \rho_j^h(d), \quad h = 1, \dots, H; j = 1, \dots, n, \quad (6a)$$

where ρ_j^h denotes the demand price for commodity h in country j . Making use now of conservation of flow equations (4), we construct equivalent demand price functions $\tilde{\rho}_j^h$; $h = 1, \dots, H$; $j = 1, \dots, n$, as follows:

$$\tilde{\rho}_j^h(Q) \equiv \rho_j^h(d). \quad (6b)$$

Transportation Cost Functions

With each link $a \in L$, and commodity h , we associate a unit transportation cost c_a^h such that

$$c_a^h = c_a^h(f), \quad \forall h, \forall a \in L. \quad (7a)$$

Because of the conservation of flow equations (2), we can define link unit transportation cost functions $\tilde{c}_a^h(Q)$, $\forall a \in L$, $\forall h$, as:

$$\tilde{c}_a^h(Q) \equiv c_a^h(f). \quad (7b)$$

The Effective Exchange Rate

Observe that, in order to appropriately quantify the effective transportation cost on a link a for a commodity h , if a commodity makes use of the link on a path from an origin country node to a destination country node, one needs to calculate the effective exchange rate associated with the commodity on link a being transported onward on path p , which is denoted by e_a^p . Note that e_a^p is the product of the exchange rates on the links on path p that include and follow link a on that path, and is given by:

$$e_a^p \equiv \begin{cases} \prod_{b \in \{a' \geq a\}_p} e_b, & \text{if } \{a' \geq a\}_p \neq \emptyset, \\ 0, & \text{if } \{a' \geq a\}_p = \emptyset, \end{cases} \quad (8)$$

where $\{a' \geq a\}_p$ denotes the set of the links including and following link a in path p , and $\emptyset$ denotes the null set.

The Effective Transportation Cost

The true transportation cost then on link a , $a \in L$, for commodity h ; $h = 1, \dots, H$, when it is used in a path p , is given by the expression:

$$\tilde{c}_a^{hp} = \tilde{c}_a^h(Q) e_a^p. \quad (9)$$

The effective transportation cost on a path, $\tilde{C}_p^h$, $\forall p \in P$, for commodity h ; $h = 1, \dots, H$, is then calculated as:

$$\tilde{C}_p^h = \sum_{a \in L} \tilde{c}_a^{hp} \delta_{ap}; \quad (10)$$

that is, the effective transportation cost on a path, which represents a trade route, is equal to the sum of the effective transportation costs for the commodity on links that make up the path.

Definition 1: The Multicommodity International Trade Equilibrium Conditions

A multicommodity path trade flow pattern $Q^* \in R_+^{nP}$ is an international trade spatial price network equilibrium pattern under subsidies and tariffs with explicit exchange rates and capacities if the following conditions hold: For all pairs of country origin and destination nodes: (i, j) ; $i = 1, \dots, m$; $j = 1, \dots, n$, and all paths $p \in P_{ij}$ as well as all commodities h ; $h = 1, \dots, H$:

$$(\tilde{\pi}_i^h(Q^*) - \text{sub}_i^h + \tau_{ij}^h) e_{ij} + \tilde{C}_p^h(Q^*) \begin{cases} \leq \tilde{\rho}_j^h(Q^*), & \text{if } Q_p^{h*} = \bar{Q}_p^h, \\ = \tilde{\rho}_j^h(Q^*), & \text{if } 0 < Q_p^{h*} < \bar{Q}_p^h, \\ \geq \tilde{\rho}_j^h(Q^*), & \text{if } Q_p^{h*} = 0. \end{cases} \quad (11)$$

Variational Inequality Formulation and Existence

Theorem 1: Variational Inequality Formulation of the Multicommodity International Trade Equilibrium Conditions

A multicommodity path trade flow pattern $Q^ \in K$, where $K \equiv \{Q | 0 \leq Q \leq \bar{Q}\}$ is a multicommodity international trade spatial price network equilibrium pattern with exchange rates and under subsidies and tariffs and capacities, according to Definition 1, if and only if it satisfies the variational inequality:*

$$\sum_{h=1}^H \sum_{i=1}^m \sum_{j=1}^n \sum_{p \in P_{ij}} \left[(\tilde{\pi}_i^h(Q^*) - \text{sub}_i^h + \tau_{ij}^h) e_{ij} + \tilde{C}_p^h(Q^*) - \tilde{\rho}_j^h(Q^*) \right] \times \left[Q_p^h - Q_p^{h*} \right] \geq 0, \quad \forall Q \in K. \quad (12)$$

Existence of an equilibrium solution Q^* is guaranteed since the feasible set K is compact.

Variational Inequality Formulation

Standard Form

Variational inequality (12) is now put into standard form (cf. Nagurney (1999)), $VI(F, \mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (13)$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$ -dimensional Euclidean space.

Specifically, we define $X \equiv Q$, $\mathcal{K} \equiv K$, and $\mathcal{N} = Hn_P$. Plus, $F(X)$ consists of the elements

$F_p^h(X) \equiv \left[(\tilde{\pi}_i^h(Q) - \text{sub}_i^h + \tau_{ij}^h) e_{ij} + \tilde{C}_p^h(Q) - \tilde{\rho}_j^h(Q) \right], \forall h, \forall i, j,$
 $\forall p \in P_{ij}$. Clearly, VI (12) can be put into standard form (13).

Illustrative Examples

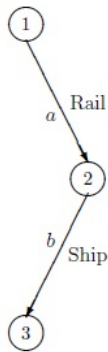
The unit of flow is a ton of wheat.

In these examples, there are no commodity path flow capacities.

These illustrative examples are stylized but, nevertheless, are grounded in real data.



Example 1 - Pre-Invasion Scenario



The network topology for Illustrative Example 1 with nodes 1 and 2 representing Ukraine and node 3 corresponding to Lebanon. There is a single path $p_1 = (a, b)$, where link a corresponds to transport to the Black Sea ports via rail inside Ukraine, and link b represents maritime transport from the Black Sea ports to Lebanon.

Example 1 - Pre-Invasion Scenario

The exchange rate data for Example 1 is drawn from early January 2022:

$$e_{13} = 55.0581, \quad e_a = 1.0000, \quad e_b = 55.0581.$$

The supply price function in Ukraine in hryvnia is:

$$\pi_1 = \pi_1(s_1) = .000136s_1 + 7,001.60.$$

The transportation cost functions in local currencies are:

$$c_a = c_a(f_a) = .000278f_a + 954.80, \quad c_b = c_b(f_b) = .000278f_b + 1,091.20.$$

The demand price function in Lebanon in Lebanese pounds is:

$$\rho_3 = \rho_3(d_3) = -.15d_3 + 602,344.00.$$

The effective exchange rates and the effective link costs are:

$$e_a^{p1} = e_a e_b = 55.0581, \quad e_b^{p1} = 55.0581,$$

$$\tilde{c}_a^{p1} = e_a^{p1} \tilde{c}_a = 55.0581 \tilde{c}_a, \quad \tilde{c}_b^{p1} = e_b^{p1} \tilde{c}_b = 55.0581 \tilde{c}_b.$$

Example 1 - Pre-Invasion Scenario

The effective path cost on path p_1 is:

$$\tilde{C}_{p_1} = \tilde{c}_a^{p_1} + \tilde{c}_b^{p_1}.$$

Applying the international trade spatial price equilibrium conditions (11), under the assumption of no tariff and no subsidy, and no quota, and assuming that $Q_{p_1}^* > 0$, we have that:

$$\tilde{\pi}_1(Q_{p_1}^*)e_{13} + \tilde{C}_{p_1}(Q_{p_1}^*) = \tilde{\rho}_3(Q_{p_1}^*),$$

which, in turn, reduces to:

$$.1881Q_{p_1}^* = 104,200.3344,$$

with solution:

$$Q_{p_1}^* = 553,962.4370.$$

The 553,962.4370 tons of wheat flow is quite reasonable, since, in 2021, Lebanon imported 520,000 tons of wheat from Ukraine, and an even greater harvest was expected in 2022.

Example 1 - Pre-Invasion Scenario

The above wheat commodity flow pattern results in a supply and a demand of:

$$s_1^* = d_3^* = Q_{p_1}^* = 553,962.4370.$$

The supply and demand prices are:

$$\pi_1(s_1^*) = \tilde{\pi}_1(Q_{p_1}^*) = 7,076.9388 \text{ UAH} = \$257.7002,$$

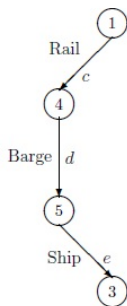
$$\rho_3(d_3^*) = \tilde{\rho}_3(Q_{p_1}^*) = 519,249.6344 \text{ LBP} = \$343.4190.$$

The incurred transportation costs are:

$$\tilde{c}_a = 1,108.8015 \text{ UAH} = \$40.3759, \quad \tilde{c}_b = 1,245.2015 \text{ UAH} = \$45.3428.$$

The supply price of \$257.7002 per ton of wheat in Ukraine (at the farmer level) and the demand price of \$343.4190 in Lebanon are close to the reported prices in 2021. Farmers in Ukraine could get about \$270 per ton of wheat before the invasion. The transportation cost pre-invasion for a ton of wheat in Ukraine to a port was about \$40, as is the result in this example.

Example 2 - Invasion Scenario



In Example 2, we consider the invasion scenario after February 24, 2022, but before the Black Sea Grain Initiative, which took effect in late July. During this period, essentially no grain was shipped from Ukraine using a Black Sea route as in Example 1. There is a single path $p_2 = (c, d, e)$. Nodes 1, 3, 4, and 5 denote Ukraine, Lebanon, Moldova, and Romania, respectively.

Example 2 - Invasion Scenario

The exchange rates for Example 2 are obtained from early July; that is, after the invasion but before the Black Sea Grain Initiative:

$$e_{13} = 51.6836, \quad e_c = .6528, \quad e_d = .2523, \quad e_e = 313.6980,$$

The exchange rates were essentially the same on July 20, 2022.

The supply price function in Ukraine in hryvnia is:

$$\pi_1(s) = \pi_1(s_1) = .002673s_1 + 2,806.30.$$

The difference in supply price function compared to the function in Example 1 is due to the damages because of the war.

Example 2 - Invasion Scenario

The transportation cost functions in local currencies are:

$$c_c = .002768f_c + 6,546.50, \quad c_d = .002172f_d + 2,324.60, \quad c_e = .000257f_e + 345.40.$$

The difference in the cost function on link c in this example and the cost function on link a in Example 1, with both entailing rail transportation in Ukraine, is due to the different rail gauges used in Ukraine and Moldova, which necessitates including loading and unloading costs. Loading and unloading costs are also accounted for in the cost function on link d .

The demand price function in Lebanon in Lebanese pounds is:

$$\rho_3(d) = \rho_3(d_3) = -.17d_3 + 793,747.50.$$

Note that due to the food security issues in Lebanon and concerns over the availability of Ukrainian wheat because of the war, the demand price function is different from the one in Example 1.

Example 2 - Invasion Scenario

According to the international trade spatial price equilibrium conditions (11), and assuming that $Q_{p_2}^* > 0$, we have that:

$$\tilde{\pi}_1(Q_{p_2}^*)e_{13} + \tilde{C}_{p_2}(Q_{p_2}^*) = \tilde{\rho}_3(Q_{p_2}^*),$$

the solution of which yields:

$$Q_{p_2}^* = 25,780.2589.$$

The wheat flow of 25,780.2589 tons is reasonable since, without access to deep-sea ports on the Black Sea, Ukraine can, at most, export around 10% of what it used to.

This commodity flow results in a supply and a demand of:

$$s_1^* = d_3^* = Q_{p_2}^* = 25,780.2589.$$

Example 2 - Invasion Scenario

The supply and demand prices are:

$$\pi_1(s_1^*) = \tilde{\pi}_1(Q^*) = 2,875.2106 \text{ UAH} = \$98.2813,$$

$$\rho_3(d_3^*) = \tilde{\rho}_3(Q^*) = 789,364.8559 \text{ LBP} = \$522.0667.$$

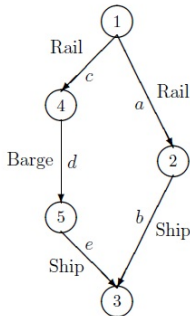
The incurred transportation link costs are:

$$\tilde{c}_c = 6,617.8597 \text{ UAH} = \$226.2137, \quad \tilde{c}_d = 2,380.5947 \text{ MDL} = \$124.6358$$

$$\tilde{c}_e = 352.0255 \text{ RON} = \$73.0358.$$

The transportation cost of grain inside Ukraine has jumped to about \$200. Furthermore, because of the ongoing war, Ukrainian farmers are earning approximately \$100 per ton of wheat, which is similar to the supply price of \$98.2813 in this example. Moreover, with the continuing food crisis in Lebanon, and, as a result of the war, the price of wheat in Lebanon has gone up to more than \$500 per ton.

Example 3 - Black Sea Grain Initiative in Place



The network topology for Example 3, where we consider the post-July 22 Black Sea Grain Initiative scenario with maritime transportation from several of the Ukrainian Black Sea ports being, again, possible. In the network above, the nodes and the links correspond to the same countries and modes of transportation as in Examples 1 and 2.

Example 3 - Black Sea Grain Initiative in Place

The exchange rates on links are from late August; that is, after the Black Sea Grain Initiative:

$$e_a = 1.0000, \quad e_b = 41.3469, \quad e_c = .5291,$$

$$e_d = .2521, \quad e_e = 309.8670, \quad e_{13} = 41.3469.$$

The exchange rates were essentially the same on July 22, 2022.

The supply price function in Ukrainian hryvnia is now:

$$\pi_1(s) = \pi_1(s_1) = .000167s_1 + 3,364.60.$$

Observe that due to the damages by the ongoing war, the supply price function, again, changes from the ones in Examples 1 and 2.

Example 3 - Black Sea Grain Initiative in Place

The transportation cost functions in local currencies are:

$$c_a = .000217f_a + 7,144.80, \quad c_b = .000246f_b + 7,423.10,$$

$$c_c = .003284f_c + 8,304.80, \quad c_d = .003097f_d + 2,397.50,$$

$$c_e = .000428f_e + 361.20.$$

The damages to the transportation infrastructure, and the congestion associated with products to be exported after the placement of the Black Sea Grain Initiative, result in different transportation cost functions from the previous examples.

The demand price function in Lebanese pounds is:

$$\rho_3(d) = \rho_3(d_3) = -.082d_3 + 796,162.50.$$

The demand price function is different from Example 2, which is in correspondence to the food security issues in Lebanon.

Example 3 - Black Sea Grain Initiative in Place

The equilibrium conditions (11) are, for this example, assuming positive commodity shipments:

$$\tilde{\pi}_1(Q^*)e_{13} + \tilde{C}_{p_1}(Q_{p_1}^*) = \tilde{\rho}_3(Q^*), \quad \tilde{\pi}_1(Q^*)e_{13} + \tilde{C}_{p_2}(Q_{p_2}^*) = \tilde{\rho}_3(Q^*).$$

The solution of the above system of equations yields a negative path flow on path p_2 , which is infeasible. Therefore, path p_2 is not used. Then, one has that: $Q_{p_1}^* = 506,566.8120$ and $Q_{p_2}^* = 0.0000$, with the commodity flows, again, in tons.

The supply is similar to what Ukraine used to export to Lebanon pre-war. With the availability of maritime transportation from Ukraine on the Black Sea, the wheat flow on path p_2 is at 0.0000, which is due to the inefficiency of transporting the grain to a Middle Eastern country by such a route and composition of modes.

Example 3 - Black Sea Grain Initiative in Place

This wheat commodity flow pattern results in the following supply and demand:

$$s_1^* = d_3^* = Q_{p_1}^* + Q_{p_2}^* = 506,566.8120,$$

with the supply and demand prices per ton now being:

$$\pi_1(s_1^*) = \tilde{\pi}_1(Q^*) = 3,449.1966 \text{ UAH} = \$94.3212,$$

$$\rho_3(d_3^*) = \tilde{\rho}_3(Q^*) = 754,624.0214 \text{ LBP} = \$499.0899.$$

The incurred transportation costs are:

$$\tilde{c}_a = 7,254.7249 \text{ UAH} = \$198.3867, \quad \tilde{c}_b = 7,547.7154 \text{ UAH} = \$206.3988,$$

$$\tilde{c}_c = 8,304.8000 \text{ UAH} = \$227.1019, \quad \tilde{c}_d = 2,397.5000 \text{ MDL} = \$123.9018,$$

$$\tilde{c}_e = 361.2000 \text{ RON} = \$74.0245.$$

Although the initiative has facilitated the transportation of wheat, the war, and nearly full storage have kept the prices high. The supply price at \$94.3212 and the demand price at \$499.0899, along with the \$206.3988 transportation cost on link *b*, reflect these issues and are preventing the demand market prices from falling.

Example 4 - Example 3 Data with Subsidy

In Example 4, we, again, consider the post-July 22 Black Sea Grain Initiative scenario with maritime transportation via the Black Sea from Ukraine possible; however, a subsidy is introduced in this example, and the impact is quantified. We consider the effect of the subsidy $sub_1 = 1,000.00$ in hryvnia on Ukrainian wheat shipped to Lebanon:

In the solution of this example, again, only path p_1 is used, and one has that: $Q_{p_1}^* = 889,408.4787$ and $Q_{p_2}^* = 0.0000$.

This wheat commodity flow pattern results in the following supply and demand:

$$s_1^* = d_3^* = Q_{p_1}^* + Q_{p_2}^* = 889,408.4787,$$

with the following supply and demand prices:

$$\pi_1(s_1^*) = \tilde{\pi}_1(Q^*) = 3,513.1312 \text{ UAH} = \$96.0696,$$

$$\rho_3(d_3^*) = \tilde{\rho}_3(Q^*) = 723,231.0047 \text{ LBP} = \$478.3273.$$

Example 4 - Example 3 Data with Subsidy

The incurred transportation costs are:

$$\tilde{c}_a = 7,337.8016 \text{ UAH} = \$200.6585, \tilde{c}_b = 7,641.8944 \text{ UAH} = \$208.9742,$$

$$\tilde{c}_c = 8,304.8000 \text{ UAH} = \$227.1019, \tilde{c}_d = 2,397.5000 \text{ MDL} = \$123.9018$$

$$\tilde{c}_e = 361.2000 \text{ RON} = \$74.0245.$$

The subsidy increases the quantity of wheat shipment, and increases the price that farmers can expect to get for a ton of wheat to \$96.0696, which is of value as the current low supply prices threaten the farmers' ability to buy seed and equipment for the next harvest season.

The subsidy also helps to reduce the demand price to \$478.3273, which can be of significant importance in countering the food crisis and associated food insecurity in Lebanon.

Modified Projection Method of Korpelevich

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $t := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \hat{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \hat{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \quad (14)$$

Step 2: Adaptation

Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + (\beta F(\hat{X}^t) - X^{t-1}), X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \quad (15)$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $t := t + 1$ and go to Step 1.

Explicit Formulae at Iteration t for the Multicommodity Product Path Flows in Step 1

The algorithm results in the following closed form expressions for (14) for the multicommodity product path flows in Step 1 for the solution of variational inequality (12):

$$\begin{aligned}\bar{Q}_p^{ht} = \max\{0, \min\{\bar{Q}_p^h, Q_p^{ht-1} + \beta(\tilde{\rho}_j^h(Q^{t-1}) - (\tilde{\pi}_i^h(Q^{t-1}) + sub_i^h - \tau_{ij}^h)e_{ij} \\ - \tilde{C}_p^h(Q^{t-1}))\}\}, \\ \forall h, \forall p.\end{aligned}\tag{16}$$

The explicit formulae for (15) in Step 2 readily follow.

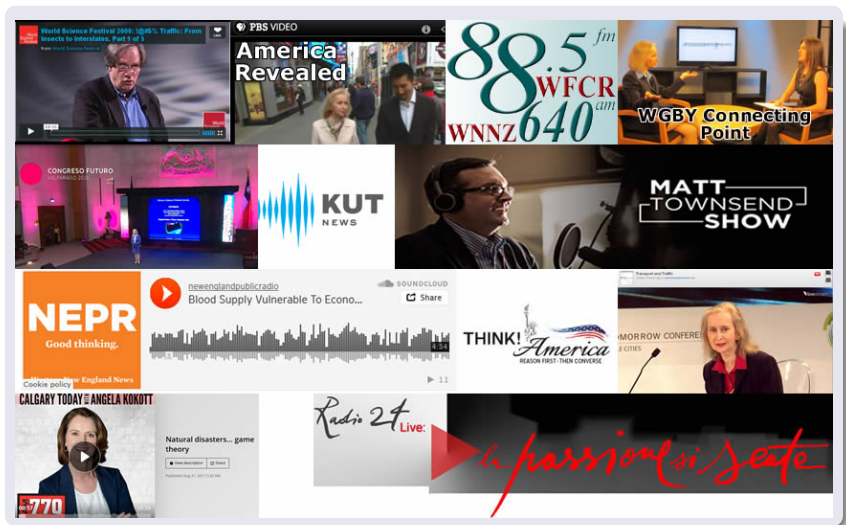
- **The importance of having alternative routes in countering disruptions and congestion** is evident in our examples.
- The results strongly confirm **the need for efficient transportation routes for trade**, as, for example, for the export of grain via maritime transport from the Black Sea ports in the case of Ukraine.
- The examples show the **benefits of subsidies for agricultural trade for both farmers and consumers**.
- **The impact of the exchange rates on the grain commodity flows and on producer and consumer prices are revealed** in the examples for different periods: pre-invasion, following the invasion, and after the Black Sea Grain Initiative.
- **The examples demonstrate the importance of the Ukrainian grain, and its relevance to global food security.**

Making a Positive Impact

Writing OpEds



Coverage by the Media



Writing OpEds in the Pandemic

On March 11, 2020 the WHO declared the pandemic. On March 12 my article on blood supply chains in *The Conversation* appeared and, on March 24 my article in *INFORMS Analytics Coronavirus Chronicles*.

THE CONVERSATION
Academic rigor. Journalistic flair.

How coronavirus is upsetting the blood supply chain

March 12, 2020 6:00am EDT



Read
Twitter
Facebook
LinkedIn
Print

The coronavirus, which causes the disease COVID-19, has created enormous anxiety, uncertainty, and disruption to our lives. Much has already been written about potential shortages of medicines and face masks, but little has been said about something only you and I can provide – lifesaving blood.

Our nation's blood supply is essential to our health care security. Blood transfusions are integral parts of major surgeries. Blood is used in the treatment of diseases, particularly sickle cell anemia and some cancers. Blood is needed for victims who have injuries caused by accidents or natural disasters. Every day, the U.S. needs 36,000 units of red blood cells, 7,000 units of platelets, and 10,000 units of plasma.

I am a professor and director of the Virtual Center for Supernetworks at the University of Massachusetts Amherst. Because of the escalating coronavirus health care crisis, I am deeply concerned the U.S. blood supply chain is under stress. The timing could hardly be worse; the COVID-19 outbreak coincides with our seasonal flu and colds.

Patients need blood in many states

Analytics
INFORMS

March 24, 2020 in Coronavirus Chronicles

The COVID-19 Pandemic and the Stressed Blood Supply Chain

By Anna Nagurney

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PRINT ARTICLE

<https://doi.org/10.1287/LVTX.2020.02.18>



Blood is essential to our nation's healthcare security. It is a life-saving product that cannot be manufactured and comes solely from volunteer donors. No substitute for blood has yet been invented. Blood transfusions are integral parts of major surgeries. Blood is a must for saving victims of accidents and natural disasters. Blood is also used in the treatment of certain diseases, including certain cancers. In the United States, 36,000 units of red blood cells are needed daily as are 7,000 units of platelets and 10,000 units of plasma. A typical donation of one pint, which can be divided into red blood cells, plasma and platelets, can save up to three lives. Adults have 8-12 pints of blood.

Even in the best of times, the complex blood supply chain in the United States is under stress. Although 38% of the U.S. population is eligible to donate blood, less than 10% actually does so in a year. Furthermore, issues of seasonality come into play with flu and colds cutting donations; the same for weather-related events and holidays. To further complicate matters, blood is perishable; platelets last five days and red blood cells have a shelf life of 42 days.

The blood banking industry, entrusted with maintaining a sufficient supply of blood, is facing a battle of the century with the COVID-19 pandemic. The timing could not be worse with this year's heavy flu and cold season, and the blood banking industry having recently undergone a massive transformation due to both economics and changes in medical procedures [1]. For example, there is increased competition among blood service organizations for donors [2]. The American Red Cross has closed some testing facilities and even eliminated mobile collection units in parts of the country. There have also been mergers and acquisitions of blood service organizations [3]. On the other hand, hospitals are now requiring less blood for certain procedures as compared to a few years ago because of changes in medical practices. This has resulted in requests for lower prices for blood from blood banks, who still have to cover costs, and some of the new costs include higher testing costs due to diseases such as Zika. And now, because of the COVID-19 pandemic, a major source of blood donations – schools – is removed.

The critical blood supply chain is unique from others that we study in operations research (O.R.) because it requires

Writing OpEds in the Pandemic

On August 4, 2020, I published an article in *The Conversation*,

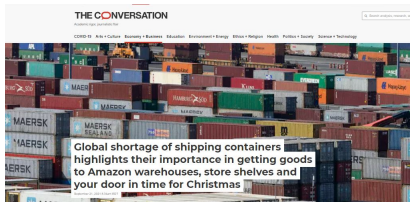
“The Raging Competition for Medical Supplies is not a Game, but Game Theory Can Help.”



On September 18, 2020, I published another article in *The Conversation*,

“Keeping Coronavirus Vaccines at Subzero Temperatures During Distribution Will Be Hard, but Likely Key to Ending Pandemic.”

Writing OpEds in the Pandemic



On April 5, 2021, I published the article,

“Today’s Global Economy Runs on Standardized Containers, as the Ever Given Fiasco Illustrates,” also in *The Conversation*.

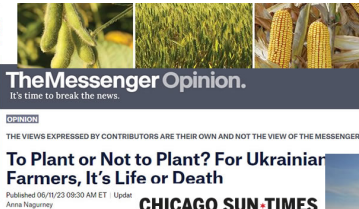
On September 21, 2021, my article,

“Global Shortage of Shipping Containers Highlights Their Importance in Getting Goods to Amazon Warehouses, Store Shelves and Your Door in Time for Christmas,” appeared in *The Conversation*. It has had over 330,000 reads.

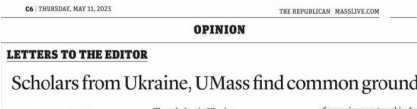
Some of My Media Interviews in the Pandemic



Writings After the Full-Scale Invasion



CHICAGO SUN•TIMES



Ukraine's Students

TO THE EDITOR:

Re "For Children of War, a Time for Play" (news article and photo essay, Aug. 8):

As children, their families and teachers get excited about the new school year throughout the world, it is imperative



Some of the Media Interviews on the War on Ukraine

Economic dangers from Russia's invasion ripple across globe

By PAUL WISEMAN and DAVID MCHUGH March 2, 2022



Feedstuffs went live.
March 16 at 2:00 p.m.



The war in Ukraine is no longer just a story about a conflict between nations. It's having an immedi...

See More



Anna Nagurney with John Moore



The John Batchelor Show

1/2: #Ukraine: The Kyiv School of Economics is open for business under fire. Paul Gregory @HooverInst @PaulR_Gregory. Anna Nagurney @Supernetworks, University of Massachusetts. Paul Becker, Duke University



Russian war in world's 'breadbasket' threatens food supply

By JOSEPH WILSON, SAMY MAGDOY, AYA BATRAWY and CHINEDU ASADU March 6, 2022

'I fear a cultural genocide'; Ukrainians in Western Mass. watch, worry and help

Published: Feb. 28, 2022, 5:55 p.m.

Threat of Russian cyber attacks likely for not just Ukraine, but also in the US

No Ikea Shelves, No Levis: The Retail Exodus From Russia Is On

Since the invasion of Ukraine began, the increasing financial and reputational risks of doing business in Russia are leading Western brands to halt operations.

Russian Sanctions Snarl Shipping Even as Pandemic Pressure Eases



March 11, 2022
Liz Aiderman and Jerry Gross

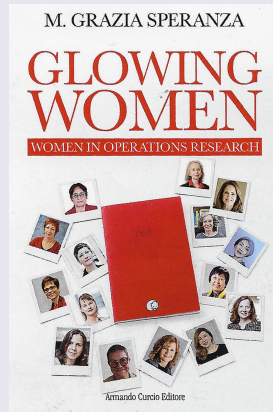
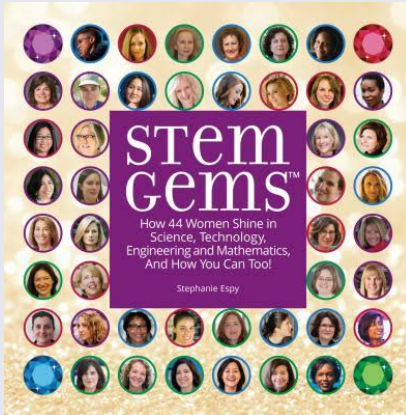
On April 22, 2020, a letter from California Attorney General Xavier Becerra to Admiral Brett Giroir, the Assistant Secretary of the US Department of Health & Human Services, and signed by US Attorney Generals of 21 other states, requested updates, because of the pandemic blood shortages, to blood donation policies that discriminate.

My March 2020 article in *The Conversation*, which was reprinted in LiveScience, was the first reference and was cited on the first page.



The Future is Bright But Investment is Needed

It is very important that we codify the history / herstory of OR for different age groups.



The Future is Bright But Investment is Needed

- The skillsets of our students are highly sought after.
- Our discipline continues to advance methodologies as well as applications. Many of our tools are used by those in different disciplines.
- Our members are regularly contacted by the media to inform and comment on current events and issues.
- The OR Society, INFORMS, IFORS, and others have done a great job in terms of advocacy and outreach.
- Quite a few of such professional society members are engaged in writing OpEds, etc.
- More are being called to advise the government (harkens to the origins of OR).
- The OR discipline arose from applications in practice, including military applications and logistics. Tighter interfaces between academics and practitioners are needed.

Thank You Very Much!



The Virtual Center for Supernetworks



Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

Director's Welcome	About the Director	Projects	Supernetworks Laboratory	Center Associates	Media Coverage	Braess Paradox
Downloadable Articles	Visuals	Audio/Video	Books	Commentaries & OpEds	The Supernetwork Sentinel	Congratulations & Kudos



Center Associates of the Virtual Center for Supernetworks

The Virtual Center for Supernetworks is an interdisciplinary center at the Isenberg School of Management that advances knowledge on large-scale networks and integrates operations research and management science, engineering, and economics. Its Director is Dr. Anna Nagurney, the Eugene M. Isenberg Chair in Integrative Studies.

Mission: The Virtual Center for Supernetworks fosters the study and application of supernetworks and serves as a resource on networks ranging from transportation and logistics, including supply chains, and the Internet, to a spectrum of economic networks.

The Applications of Supernetworks Include: decision-making, optimization, and game theory; supply chain management; critical infrastructure from transportation to electric power networks; financial networks; knowledge and social networks; energy, the environment, and sustainability; cybersecurity; Future Internet Architectures; risk management; network vulnerability, resiliency, and performance metrics; humanitarian logistics and healthcare.

Announcements and Notes	Photos of Center Activities	Photos of Network Innovators	Friends of the Center	Course Lectures	Fulbright Lectures	UMass Amherst INFORMS Student Chapter
Professor Anna Nagurney's Blog	Network Classics	Doctoral Dissertations	Conferences	Journals	Societies	Archive

More information on our work can be found on the
Supernetwork Center site:

<https://supernet.isenberg.umass.edu/>